#### Storm Surge Predictions from Ocean- to Subgrid-Scales

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#### Storm Surge Modeling

ADvanced CIRCulation (ADCIRC) solves modified forms of the shallow water equations ...

We use ADCIRC to represent the long waves of tides and storm surge

– Solves the generalized wave continuity equation (GWCE) for water levels ( $\zeta$ ):

$$\frac{\partial^{2}\zeta}{\partial t^{2}} + \tau_{0}\frac{\partial\zeta}{\partial t} + \frac{\partial\tilde{J}_{x}}{\partial x} + \frac{\partial\tilde{J}_{y}}{\partial y} - UH\frac{\partial\tau_{0}}{\partial x} - VH\frac{\partial\tau_{0}}{\partial y} = 0$$

- Solves the depth-averaged momentum equations for currents (U, V):

$$\frac{\mathrm{D}U}{\mathrm{D}t} - fV = -g\frac{\partial}{\partial x}\left[\zeta + \frac{p_s}{g\rho_0} - \alpha\eta\right] + \frac{\tau_{sx} + \tau_{bx}}{\rho_0 H} + \frac{M_x - D_x}{H}$$

$$\frac{\mathrm{D}V}{\mathrm{D}t} + fU = -g\frac{\partial}{\partial y}\left[\zeta + \frac{p_s}{g\rho_0} - \alpha\eta\right] + \frac{\tau_{sy} + \tau_{by}}{\rho_0 H} + \frac{M_y - D_y}{H}$$

Storm Surge Modeling

... and ADCIRC uses high-resolution in space and time

In geographic space:

- Piecewise-linear, continuous, Galerkin finite elements
  - Unique values for  $(\zeta, U, V)$  at every mesh vertex
- Typical minimum mesh spacings of 10 to 50 m

In time:

- Semi-implicit
  - Implicit solution of GWCE using Jacobi Conjugate Gradient (JCG) solver
  - Explicit solution of momentum equations with lumped mass matrix
- Fully explicit
  - Also possible to use lumped mass matrix for solution of GWCE
- Typical time steps of 0.5 to 10 sec



Storm Surge Modeling Coastal NC has a wide range of spatial scales ...



Storm Surge Modeling ... which we can explore by zooming to the Neuse River Estuary ...

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.. which has a main estuary, smaller channels, floodplains, etc. ...



Storm Surge Modeling ... and this complexity is represented in the DEMs

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Storm Surge Modeling If we are not careful with how we design our mesh ...

Storm Surge Modeling .... then we may alias the smaller features ...

Storm Surge Modeling ... but there are trade-offs with higher resolution

Subgrid Corrections We want to have our cake ...



# Subgrid Corrections ... and eat it, too



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Subgrid Corrections

Long history of subgrid corrections for shallow water flows ...

*Subgrid corrections* use information at smaller scales to 'correct' flow variables (water levels, current velocities) at the model scale

Selected applications to shallow water flows:

- Defina (2000) corrected advection and partially wet cells
  - $\rightarrow\,$  Able to coarsen by factor of 32
- Casulli (2009) and Casulli and Stelling (2011) also corrected partially wet cells
  - $\rightarrow\,$  Used lookup tables created from high-resolution elevation data
- Volp (2013) corrected bottom stress
  - $\rightarrow\,$  Improved discharge and water surface slope relative to high-resolution counterparts

Able to coarsen the model resolution and still represent small-scale flow pathways and barriers

 $\rightarrow\,$  Higher accuracy at same resolution, higher efficiency at coarser resolution

Subgrid Corrections

.. and we used subgrid corrections to improve connectivity for ADCIRC



#### Subgrid Corrections in ADCIRC

Averaged Equations Closure Terms Look-up Tables

## Benefits on Synthetic Test Cases Level 0 (Wet Fraction) Level 1 (Advection and Bottom Friction)

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#### Implementation in ADCIRC requires careful definitions of averaging areas







Govering equations are averaged to the model scale ...

A given flow variable Q can be averaged, e.g. Kennedy *et al.* (2019):

- To the grid/mesh scale:

$$\langle Q \rangle_G \equiv rac{1}{A_G} \iint_{A_W} Q \, \mathrm{d} A$$

- To only the wet part of the grid/mesh scale:

$$\langle Q \rangle_W \equiv rac{1}{A_W} \iint_{A_W} Q \; \mathrm{d}A$$

- Where the areas are related by:

$$A_W = \phi A_G$$

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... and we can simplify the time derivatives ...

We can use rules from Whitacker (1985) to interchange differentiation

- We assume away the boundary integrals

For example, to average a time derivative:

$$\begin{split} \langle \frac{\partial UH}{\partial t} \rangle_{G} &= \frac{1}{A_{G}} \iint_{A_{W}} \frac{\partial UH}{\partial t} \, \mathrm{d}A \\ &= \frac{1}{A_{G}} \frac{\partial}{\partial t} \iint_{A_{W}} UH \, \mathrm{d}A - \frac{1}{A_{G}} \iint_{\Gamma_{W}} UH \left(U_{B} \cdot \mathbf{n}_{S}\right) \, \mathrm{d}S \\ &= \frac{\partial \langle UH \rangle_{G}}{\partial t} \end{split}$$

because we assume H = 0 on the wet/dry boundary  $\Gamma_W$ 

... and we can simplify the spatial derivatives ...

The averaging starts similarly for a spatial derivative:

$$\begin{split} \langle \frac{\partial UUH}{\partial x} \rangle_{G} &= \frac{1}{A_{G}} \iint_{A_{W}} \frac{\partial UUH}{\partial x} \, \mathrm{d}A \\ &= \frac{1}{A_{G}} \frac{\partial}{\partial x} \iint_{A_{W}} UUH \, \mathrm{d}A + \frac{1}{A_{G}} \iint_{\Gamma_{W}} UUH \, \mathrm{n}_{s,x} \, \mathrm{d}S \\ &= \frac{\partial \langle UUH \rangle_{G}}{\partial x} \end{split}$$

but then we also introduce a closure term:

$$\frac{\partial \langle UUH \rangle_G}{\partial x} = \frac{\partial C_{UU} \langle U \rangle \langle UH \rangle_G}{\partial x}$$

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 $\ldots$  to derive the averaged governing equations for ADCIRC

We apply these averaging rules to every term in the governing equations

- Example of momentum conservation in x-direction:

$$\frac{\partial \langle UH \rangle_{G}}{\partial t} + \frac{\partial C_{UU} \langle U \rangle \langle UH \rangle_{G}}{\partial x} + \frac{\partial C_{VU} \langle V \rangle \langle UH \rangle_{G}}{\partial y} - f \langle VH \rangle_{G}$$
$$= -g C_{\zeta} \langle H \rangle_{G} \frac{\partial \langle \zeta \rangle_{W}}{\partial x} - g \langle H \rangle_{G} \frac{\partial P_{A}}{\partial x} + \phi \langle \frac{\tau_{sx}}{\rho_{0}} \rangle_{W}$$
$$- C_{M,f} \frac{|\langle \mathbf{U} \rangle| \langle UH \rangle_{G}}{\langle H \rangle_{W}} + \frac{\partial}{\partial x} \tilde{E}_{h} \frac{\partial \langle UH \rangle_{G}}{\partial x} + \frac{\partial}{\partial y} \tilde{E}_{h} \frac{\partial \langle UH \rangle_{G}}{\partial y}$$

in which the red coefficients are new closure terms

- Similarly for momentum conservation in y-direction, mass conservation

#### **Closure Terms**

We can assign closures with levels of complexity  $\ldots$ 

We used 'Level 0' and 'Level 1' closures:

		Conventional	Level 0	Level 1
Wet/dry	$\phi$	0 or 1	$A_W/A_G$	$A_W/A_G$
Advection	$C_{UU}, C_{VU}, C_{UV}, C_{VV}$	1	1	$rac{1}{\langle H  angle_W} \langle H^2/C_f  angle_W R_v^2$
Friction	$C_{M,f}$	$C_f = gn^2/H^{1/3}$	$\langle C_f \rangle_W$	$\langle H \rangle_W R_v^2$
Surface gradient	$C_{\zeta}$	1	1	1

Note the differences for the wet/dry status, advection, and friction terms

- Level 0 only changes the wet/dry status to allow partially wet cells/elements
- Level 1 adds corrections for advection and friction

#### **Closure Terms**

... and these closures allow for partially wet elements/areas

Level-0 closures required a major revision to ADCIRC's wet/dry algorithm

 $\rightarrow\,$  Removed extensive logic to compare water levels, velocities between vertices



Now the status is determined solely by the total water depth:

$$\langle H 
angle_{G} > \langle H 
angle_{G_{min}} = 0.1 \,\,\mathrm{m}$$

Look-up Tables

Closures and averaged values can be pre-computed and stored in look-up tables ...

Following variables depend on subgrid information:

- Elements:  $\langle H \rangle_{G}$ ,  $C_{UU}$ ,  $C_{VU}$ ,  $C_{UV}$ ,  $C_{VV}$ ,  $\phi$
- Vertices:  $\langle H \rangle_G$ ,  $\langle H \rangle_W$ ,  $C_{M,f}$ ,  $\phi$

We can pre-compute these variables:

- Pick a range of possible water levels, e.g.  $\langle \zeta \rangle_W = -5$  to 5 m
- For each possible  $\langle \zeta \rangle_W$ , compute other variables based on high-resolution elevation and landcover raster datasets
- Store variables in look-up tables for use during the simulation

We reduced file sizes by using a range of possible wet-area fractions,  $\phi = 0$  to 1

#### Look-up Tables ... by using an open-source subgrid calculator

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### Subgrid Corrections in ADCIRC

Averaged Equations Closure Terms Look-up Tables

## Benefits on Synthetic Test Cases Level 0 (Wet Fraction) Level 1 (Advection and Bottom Friction)

Expansion to Storm Surge on Ocean Scales Matthew (2016) in South Atlantic Bight

Conclusions



#### Level 0 (Wet Fraction) Partially wet elements are important ...

Consider a winding channel:

- Channel width 250 m and depth 1 m below surrounding topography
- Meshes: coarse (1000 m) and fine (minimum 10 m)
- Tides from south boundary with amplitude 1 m  $\,$



Level 0 (Wet Fraction) ... because they allow tides to propagate further into the channel



Level 1 (Advection and Bottom Friction) Higher-level closures are sometimes important ...

Consider another winding channel:

- Channel width 5 m and depth 1 m below surrounding
- Meshes: coarse (24 m) and fine (minimum 5 m)
- Constant flow by specifying water depths



Level 1 (Advection and Bottom Friction) ... depending on the overall flow depth ...



Level 1 (Advection and Bottom Friction) ... and the local flow velocities

Velocity magnitude differences due to:

- Advection and bottom friction
- Only bottom friction
- Only advection



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#### Matthew (2016) in South Atlantic Bight Widespread flooding along South Atlantic Bight (SAB)



# $\begin{array}{l} \mbox{Matthew (2016) in South Atlantic Bight} \\ \mbox{Elevation and landcover described by 830 datasets and 197 GB} \end{array}$



#### Matthew (2016) in South Atlantic Bight 'Forecast-grade' mesh with 770K vertices and minimum resolution of 500 m



#### Matthew (2016) in South Atlantic Bight Flooding extents are similar to SACS mesh that is 15 times larger



#### Matthew (2016) in South Atlantic Bight Improved connectivity to far-inland regions like New Bern NC



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#### Matthew (2016) in South Atlantic Bight Flooding at more locations, and better match to observed peaks



Matthew (2016) in South Atlantic Bight Subgrid ADCIRC has overhead, but offers significant speed-ups

# Wall-Clock Time (CPU-hr)SACS Conventional5860SABv2 Conventional386SABv2 Subgrid433

Wall-Clock Time RatioSABv2 Subgrid / SABv2 Conventional1.12SACS Conventional / SABv2 Subgrid13.55

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#### Conclusions Subgrid ADCIRC

The main contributions of this research are:

- 1. Subgrid corrections were added to ADCIRC
  - $\rightarrow\,$  Hurricane-strength forcing on ocean domains
- 2. Increases in accuracy and hydraulic connectivity on coarsened meshes
  - $\rightarrow$  Flooding to more locations in South Atlantic Bight, better match to observations during Matthew (2016)
- 3. Efficiency gains on coarsened meshes
  - $\rightarrow\,$  Speed-ups by factors of 13+

Future efforts should focus on:

- Optimizing code to reduce overhead
- New applications



#### Conclusions

#### Recent manuscripts describe Johnathan's PhD research

	Ocean Modelling 167 (2021) 101887	10.1007/s11069-023-05975-2		
	Contents lists available at ScienceDirect Ocean Modelling	Natural Hazards https://doi.org/10.1007/s11069-023-05975-2		
ELSEVIER	journal homepage: www.alsevier.com/locate/ocemod	ORIGINAL PAPER		
Subgrid corrections in finite-element modeling of storm-driven coas flooding		Storm surge predictions from ocean to subgrid scales		
Johnathan L. Woodruff <sup>1</sup> <sup>1</sup> , <sup>1</sup> , J.C. Dietrich <sup>1</sup> , D. Wirasaet <sup>1</sup> , A.B. Kennedy <sup>1</sup> , D. Bolster <sup>1</sup> , Z S.D. Medlin <sup>1</sup> , R.L. Kolar <sup>2</sup> <sup>1</sup> Papermen of Civil Contention of Invironmental Engineering, North Cardina Sinte University, 2501 Sinton Drive, Raligh, NC, 27607, <sup>1</sup> Paperment of Civil and Environmental Engineering and Earth Science, University of Neur. Dums, South Intel, RK, 46556, Unitel Status of A <sup>1</sup> School of Civil Engineering and Environmental Science, University of Oklahoms, Norman, OK, 73019, United Status of America		Johnathan Woodruff <sup>1</sup> • J. C. Dietrich <sup>1</sup> · D. Wirasaet <sup>2</sup> · A. B. Kennedy <sup>2</sup> · D. Bolster <sup>2</sup> Received: 25 October 2022 / Accepted: 30 March 2023 © The Author(s), under exclusive licence to Springer Nature B.V. 2023		
A R T I C L E I N Keyword: Storm surge Subgrid ADCIRC Wetting and drying Shallow water equations	A B S T R A C T Coatral flooding models are used to predict the timing and magnitude for all time forecasting and large term design. However, there is a word's represent flow pathways and barriers at the scales of critical infrastrue subprid corrections, which use information at multiles accels to 'correct' violocitiea) averaged over the mesh scale. Recent studies have shown a of magnitude, with the ability to decrease further if the model times the in a synthetic domain and a small have in Magnitude corrections, is et in a synthetic domain and a small have in Magnitude corrections, is es and storm sarge in southwest. Louisians. In these tests we observed the model speed by 10 to 50 times, while still representing flow through d locations.	Abstract The inland propagation of storm surge caused by tropical cyclones depends on large and small waterways to connect the open ocean to inland bays, estuaries, and floodplains. Numerical models for storm surge require these waterways and their surrounding topogra- phy to be resolved sufficiently, which can require millions of computational cells for flood- ing simulations on a large (ocean scale) computational domain, leading to higher demands for computational resources and longer wall-clock times for simulations. Alternatively, the governing shallow water equations can be modified to introduce subgrid corrections that allow coarser and cheaper simulations with comparable accuracy. In this study, subgrid		
10.1016/	i.ocemod.2021.101887	corrections are extended for the first time to simulations at the ocean scale. Higher-level		