Improving predictions of coastal flooding via sub-mesh corrections

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Introduction

- High resolution is computationally **costly**
- Hinders the speed of ADCIRC runs
- Delays the forecast predictions

This study aims to **increase the accuracy and efficiency of ADCIRC** by:

- Adding sub-mesh correction factors to the governing equations
- Running on coarsened meshes
1. High resolution bathymetry surrounding Delacroix, LA

2. NOMAD mesh v1e MSL (HSOFS)
   - This mesh is used in real-time forecasting by NOAA and the ADCIRC Prediction System (APS).

3. Interpolated bathymetry of the mesh
Sub-mesh Features

- Sub-mesh features:
  - Hydraulic features that influence flow
  - Exist below the resolution of the mesh
  - Include: small scale channels, ponds, marsh grasses, and roadways
Theory

- The primitive shallow water equations were first averaged using techniques outlined in Kennedy et al. 2019.
- These averaged primitive equations were then transformed into the GWCE and conservative momentum equations ADCIRC uses.
Averaged Variables Theory

• To obtain the averaged variables we integrate inside each element.

• A given dummy variable $Q$ would be averaged as follows:

\[
\langle Q \rangle_G \equiv \frac{1}{A_G} \iiint_{A_W} Q \, dA \quad \& \quad \langle Q \rangle_W \equiv \frac{1}{A_W} \iiint_{A_W} Q \, dA \quad \text{Where: } A_W = \phi A_G
\]
GWCE Conversion

\[
\frac{\partial^2 \zeta}{\partial t^2} + \tau_0 \frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{J}_x}{\partial x} + \frac{\partial \tilde{J}_y}{\partial y} - UH \frac{\partial \tau_0}{\partial x} - VH \frac{\partial \tau_0}{\partial y} = 0
\]

Where:

- \( \phi \) is the wet area fraction
- Subscripts \( W \) and \( G \) mean the variable was averaged over the wet area or whole area respectively

\[
\phi \frac{\partial^2 \langle \zeta \rangle_W}{\partial t^2} + \phi \tau_0 \frac{\partial \langle \zeta \rangle_W}{\partial t} + \frac{\partial \langle \tilde{J}_x \rangle_W}{\partial x} + \frac{\partial \langle \tilde{J}_y \rangle_W}{\partial y} - \langle U \rangle_W \langle H \rangle_G \frac{\partial \tau_0}{\partial x} - \langle V \rangle_W \langle H \rangle_G \frac{\partial \tau_0}{\partial y} = 0
\]
GWCE Conversion

\[
\tilde{J}_x = - \frac{\partial \langle U \rangle_W \langle U \rangle_W \langle H \rangle_G}{\partial x} - \frac{\partial \langle V \rangle_W \langle U \rangle_W \langle H \rangle_G}{\partial y} + f \langle V \rangle_W \langle H \rangle_G - g \langle h \rangle_G \frac{\partial \langle \zeta \rangle_W}{\partial x} - \frac{\phi g \partial \langle \zeta \rangle_W^2}{2 \partial x} \\
+ \phi \left( \frac{\tau_{sx}}{\rho_0} \right)_W - \frac{C_f \langle U \rangle_W \langle U \rangle_W \langle H \rangle_G}{\langle H \rangle_W} + \frac{\partial}{\partial x} \left( E_h \frac{\partial \langle U \rangle_W \langle H \rangle_G}{\partial x} \right) + \frac{\partial}{\partial y} \left( E_h \frac{\partial \langle U \rangle_W \langle H \rangle_G}{\partial y} \right) + \tau_0 \langle U \rangle_W \langle H \rangle_G
\]

\[
\tilde{J}_y = - \frac{\partial \langle U \rangle_W \langle V \rangle_W \langle H \rangle_G}{\partial x} - \frac{\partial \langle V \rangle_W \langle V \rangle_W \langle H \rangle_G}{\partial y} - f \langle U \rangle_W \langle H \rangle_G - g \langle h \rangle_G \frac{\partial \langle \zeta \rangle_W}{\partial y} - \frac{\phi g \partial \langle \zeta \rangle_W^2}{2 \partial y} \\
+ \phi \left( \frac{\tau_{sy}}{\rho_0} \right)_W - \frac{C_f \langle U \rangle_W \langle V \rangle_W \langle H \rangle_G}{\langle H \rangle_W} + \frac{\partial}{\partial x} \left( E_h \frac{\partial \langle V \rangle_W \langle H \rangle_G}{\partial x} \right) + \frac{\partial}{\partial y} \left( E_h \frac{\partial \langle V \rangle_W \langle H \rangle_G}{\partial y} \right) + \tau_0 \langle V \rangle_W \langle H \rangle_G
\]
Sub-mesh calculations

- Sub-mesh calculations are performed using a high resolution digital elevation model (DEM) underlying a ADCIRC mesh of coarser resolution.
- Sub-mesh correction factors such as $\phi$, $\langle H \rangle_G$, and $\langle h \rangle_G$ are found by integrating the sub-areas within each element.
- Sub-mesh quantities are pre-computed and read into ADCIRC.
Sub-mesh calculations

- Within the elemental loops of the GWCE and Momentum codes averaged variables are looked up based on water surface elevation at each vertex.
- Averaged variables are also stored in vertex arrays to be used in wetting and drying and the nodal loops of the momentum code.
Sub-mesh in 2D

- In 2D the elements are split up into 3 sub-areas.
- Sub-mesh calculations are performed based on the DEM lying beneath the mesh for each sub-area within the element.

- Green = WET
- Yellow = PARTIALLY WET
- Red = DRY
• Divide element into 3 sub-areas based on the halfway points between vertices.

• Divide each sub-area into 2 triangles

• Refine each triangle into many smaller triangles that are roughly the same resolution as the sub-mesh

• Apply DEM data to the smaller triangles

• Average sub-mesh variables over each sub-area
Wetting and Drying

REALITY  TRADITIONAL ADCIRC  SUB-MESH ADCIRC

$H_{\text{min}}$
Wetting and Drying Algorithm

- A totally new wetting and drying algorithm was used to replace the existing.
- The wet/dry state of each vertex is determined based on the wet area surrounding the vertex.
- Elemental wet/dry state is determined by the average wet area within each element.
Wetting and Drying Algorithm

BEGIN
Wetting and Drying

ADJUST
ζ of DRY vertices to the ζ of the neighboring WET vertices

COMPUTE
averaged variables \( \phi, \langle H \rangle_G, \langle H \rangle_W \)

EVALUATE
wet/dry state \( \phi \geq \phi_{dry} \)

END
Wetting and Drying
Discretization

• The discretization of the governing equations mimics the Conservative Version 1 discretization.

• Conservative Version 1 assumes the un-differentiated terms are elementally averaged and uses a linear expansion in space for the conservative flux terms.
Results

• Sub-mesh ADCIRC was run on a coarsened mesh to demonstrate its ability to accurately predict water surface elevations.
• Traditional ADCIRC was run on both a coarsened and high resolution mesh as a baseline to compare sub-mesh results.
Model Setup

- DT = 1 second
- $\tau_0 = 0.005$
- FFACTOR = 0.0025
- ESLM = 2.0
- No ramp
- 1 day runtime
- Sub-mesh $\phi_{dry} = 0.25$
- 2 m diurnal tidal signal was used to force the model
• High resolution (10 m) DEM featuring a 250 m wide winding channel was used to calculate the sub-mesh variables.
• High resolution mesh was created using this DEM.
• This mesh resolves the small channel.

Vertices: 6420    Elements: 12742
• Coarse resolution mesh was created using the DEM.
• This mesh was created by simply paving elements over the domain.

Vertices: 192  Elements: 334
High Resolution

Vertices: 6420    Elements: 12742

Coarse Resolution

Vertices: 192    Elements: 334
Water surface elevations were taken from these vertices.
The sub-mesh adds roughly 40% to computation time. This is likely to improve as the code is clean up for use on more realistic test cases.
Conclusion

• The sub-mesh code was able to accurately predict water surface elevation on a coarsened mesh.
• The computational cost of the sub-mesh model was close to 30 times less than the high resolution traditional run.
• Coarse mesh creation was extremely fast and easy compared to creating the high resolution mesh.
Future Work

• The wetting and drying routine is still being adjusted to allow for smaller $\phi_{dry}$.
• A threshold of around 5% would be ideal in the future.
• The model is also experiencing stability problems at the top of the domain when running multi-day simulations.
• Testing on a realistic domain such as Caernarvon Marsh, LA is soon to come.
Thank you!

Questions?

Thank you to the National Science Foundation for sponsoring this research.