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Improving predictions of coastal flooding via sub-mesh corrections

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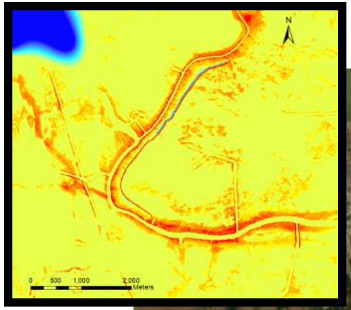
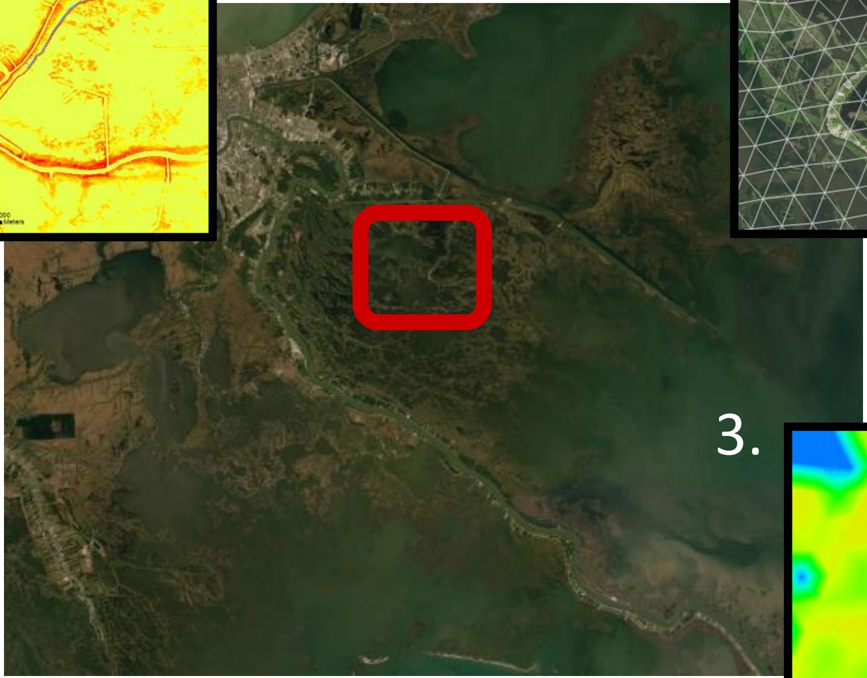
Introduction

- High resolution is computationally **costly**
    - Hinders the speed of ADCIRC runs
    - Delays the forecast predictions
- This study aims to **increase the accuracy and efficiency of ADCIRC** by:
    - Adding sub-mesh correction factors to the governing equations
    - Running on coarsened meshes

1.

High resolution bathymetry surrounding Delacroix, LA





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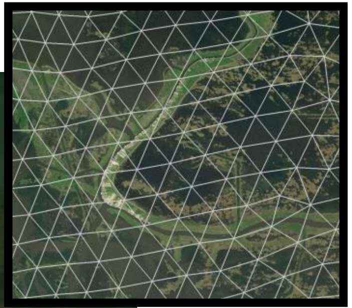
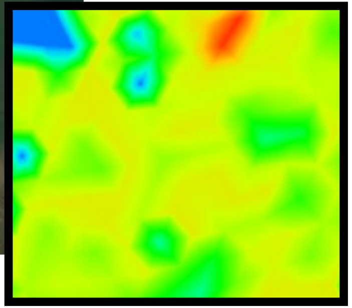


NOMAD mesh v1e MSL (HSOFS)

- This mesh is used in real-time forecasting by NOAA and the ADCIRC Prediction System (APS).

3.

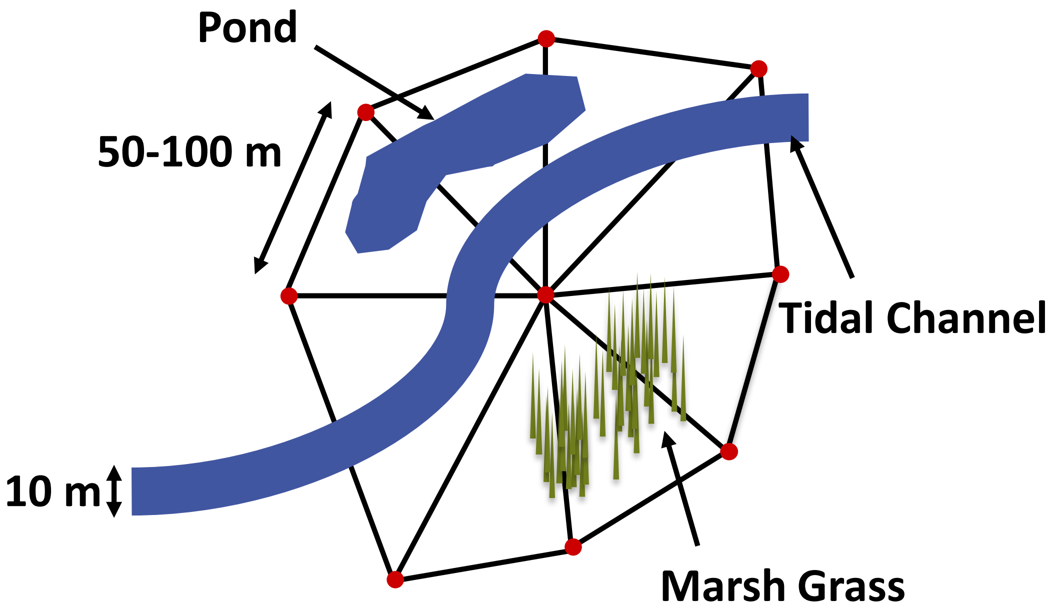


Interpolated bathymetry of the mesh

Caernarvon Marsh, LA

Sub-mesh Features

- Sub-mesh features:
  - Hydraulic features that influence flow
  - Exist below the resolution of the mesh
  - Include: small scale channels, ponds, marsh grasses, and roadways



Theory

- The primitive shallow water equations were first averaged using techniques outlined in Kennedy et al. 2019.
- These averaged primitive equations were then transformed into the GWCE and conservative momentum equations ADCIRC uses.

Averaged Variables Theory

- To obtain the averaged variables we integrate inside each element.
- A given dummy variable Q would be averaged as follows:

$$\langle Q \rangle_G \equiv \frac{1}{A_G} \iint_{A_W} Q dA \qquad \& \qquad \langle Q \rangle_W \equiv \frac{1}{A_W} \iint_{A_W} Q dA \qquad \text{Where: } A_W = \phi A_G$$

GWCE Conversion

$$\frac{\partial^2 \zeta}{\partial t^2} + \tau_0 \frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{J}_x}{\partial x} + \frac{\partial \tilde{J}_y}{\partial y} - UH \frac{\partial \tau_0}{\partial x} - VH \frac{\partial \tau_0}{\partial y} = 0$$

$$\Downarrow$$

$$\phi \frac{\partial^2 \langle \zeta \rangle_W}{\partial t^2} + \phi \tau_0 \frac{\partial \langle \zeta \rangle_W}{\partial t} + \frac{\partial \langle \tilde{J}_x \rangle_W}{\partial x} + \frac{\partial \langle \tilde{J}_y \rangle_W}{\partial y} - \langle U \rangle_W \langle H \rangle_G \frac{\partial \tau_0}{\partial x} - \langle V \rangle_W \langle H \rangle_G \frac{\partial \tau_0}{\partial y} = 0$$

Where:

- ϕ is the wet area fraction
- Subscripts $\langle \quad \rangle_W$ and $\langle \quad \rangle_G$ mean the variable was averaged over the wet area or whole area respectively

GWCE Conversion

$$\langle \tilde{J}_x \rangle_W = -\frac{\partial \langle U \rangle_W \langle U \rangle_W \langle H \rangle_G}{\partial x} - \frac{\partial \langle V \rangle_W \langle U \rangle_W \langle H \rangle_G}{\partial y} + f \langle V \rangle_W \langle H \rangle_G - g \langle h \rangle_G \frac{\partial \langle \zeta \rangle_W}{\partial x} - \phi \frac{g}{2} \frac{\partial \langle \zeta \rangle_W^2}{\partial x}$$

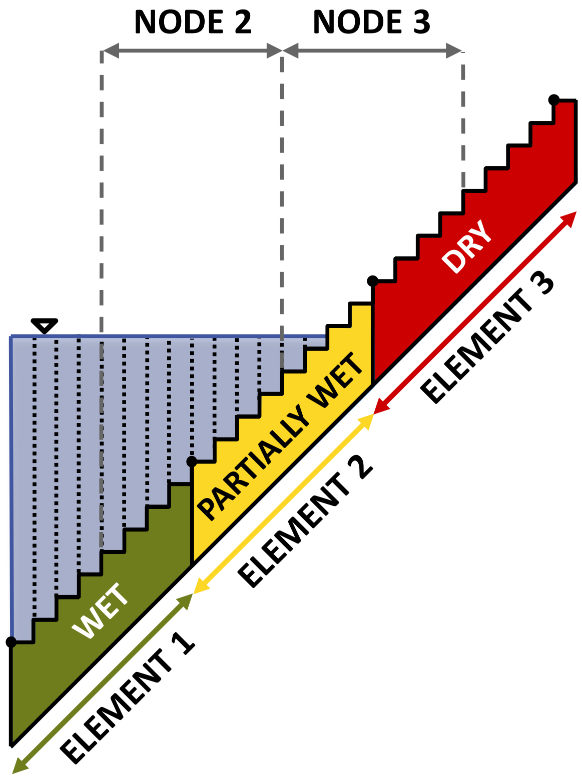
$$+\phi \left\langle \frac{\tau_{sx}}{\rho_0} \right\rangle_W - \frac{C_f |\langle \boldsymbol{U} \rangle_W| \langle U \rangle_W \langle H \rangle_G}{\langle H \rangle_W} + \frac{\partial}{\partial x}\left(E_h \frac{\partial \langle U \rangle_W \langle H \rangle_G}{\partial x} \right) + \frac{\partial}{\partial y}\left(E_h \frac{\partial \langle U \rangle_W \langle H \rangle_G}{\partial y} \right) + \tau_0 \langle U \rangle_W \langle H \rangle_G$$

$$\langle \tilde{J}_y \rangle_W = -\frac{\partial \langle U \rangle_W \langle V \rangle_W \langle H \rangle_G}{\partial x} - \frac{\partial \langle V \rangle_W \langle V \rangle_W \langle H \rangle_G}{\partial y} - f \langle U \rangle_W \langle H \rangle_G - g \langle h \rangle_G \frac{\partial \langle \zeta \rangle_W}{\partial y} - \phi \frac{g}{2} \frac{\partial \langle \zeta \rangle_W^2}{\partial y}$$

$$+\phi \left\langle \frac{\tau_{sy}}{\rho_0} \right\rangle_W - \frac{C_f |\langle \boldsymbol{U} \rangle_W| \langle V \rangle_W \langle H \rangle_G}{\langle H \rangle_W} + \frac{\partial}{\partial x}\left(E_h \frac{\partial \langle V \rangle_W \langle H \rangle_G}{\partial x} \right) + \frac{\partial}{\partial y}\left(E_h \frac{\partial \langle V \rangle_W \langle H \rangle_G}{\partial y} \right) + \tau_0 \langle V \rangle_W \langle H \rangle_G$$

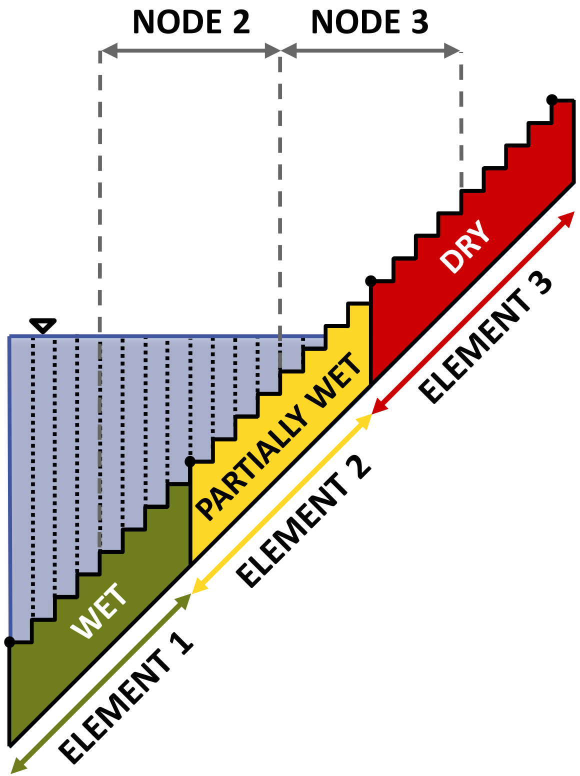
Sub-mesh calculations

- Sub-mesh calculations are performed using a high resolution digital elevation model (DEM) underlying a ADCIRC mesh of coarser resolution.
- Sub-mesh correction factors such as ϕ, $\langle H \rangle_G$, and $\langle h \rangle_G$ are found by integrating the sub-areas within each element.
- Sub-mesh quantities are pre-computed and read into ADCIRC.



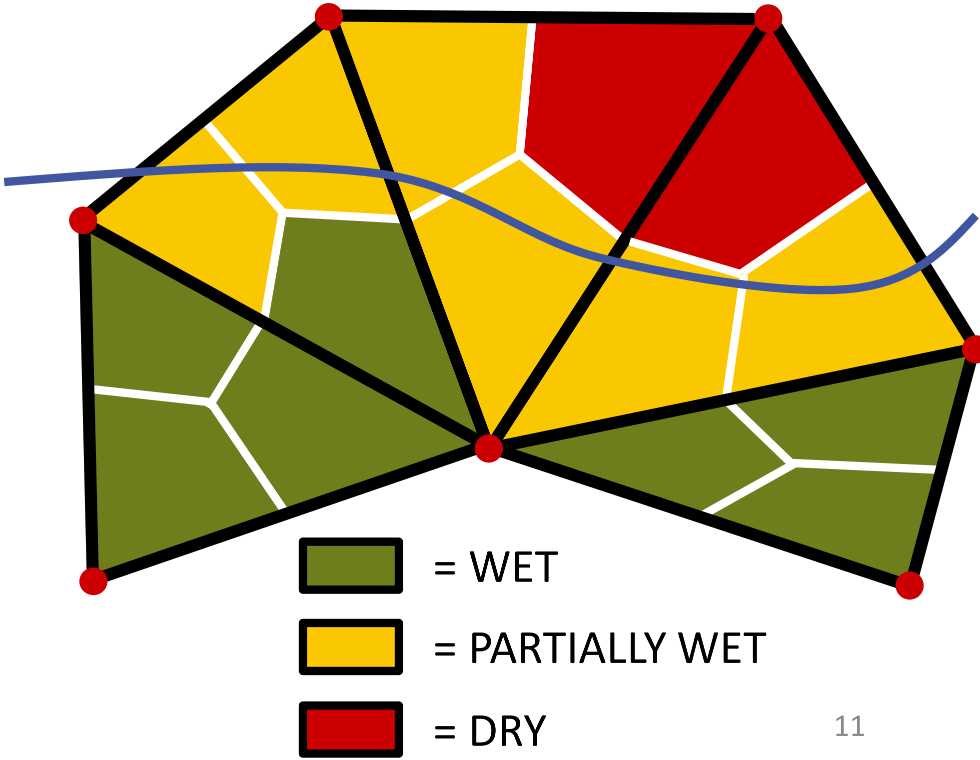
Sub-mesh calculations

- Within the elemental loops of the GWCE and Momentum codes averaged variables are looked up based on water surface elevation at each vertex.
- Averaged variables are also stored in vertex arrays to be used in wetting and drying and the nodal loops of the momentum code.

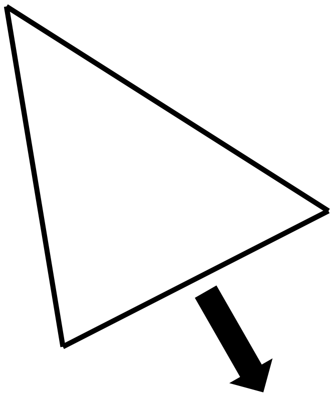


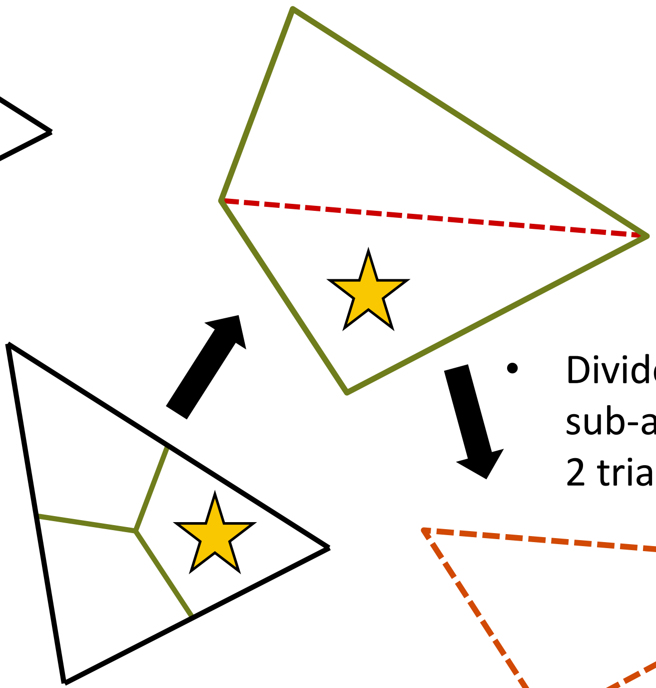
Sub-mesh in 2D

- In 2D the elements are split up into 3 sub-areas.
- Sub-mesh calculations are performed based on the DEM lying beneath the mesh for each sub-area within the element.



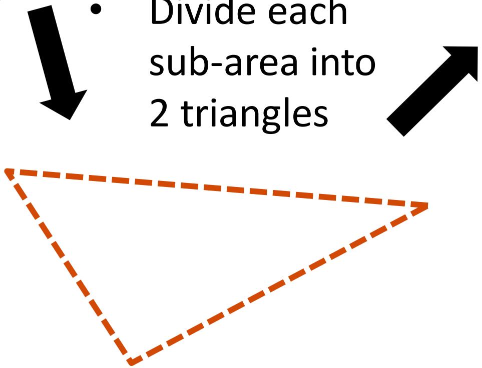
Sub-mesh in 2D

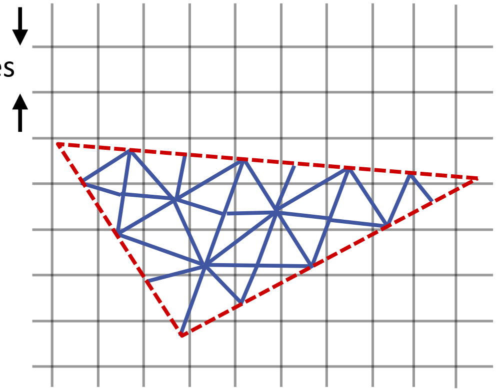




- Divide element into 3 sub-areas based on the halfway points between vertices.

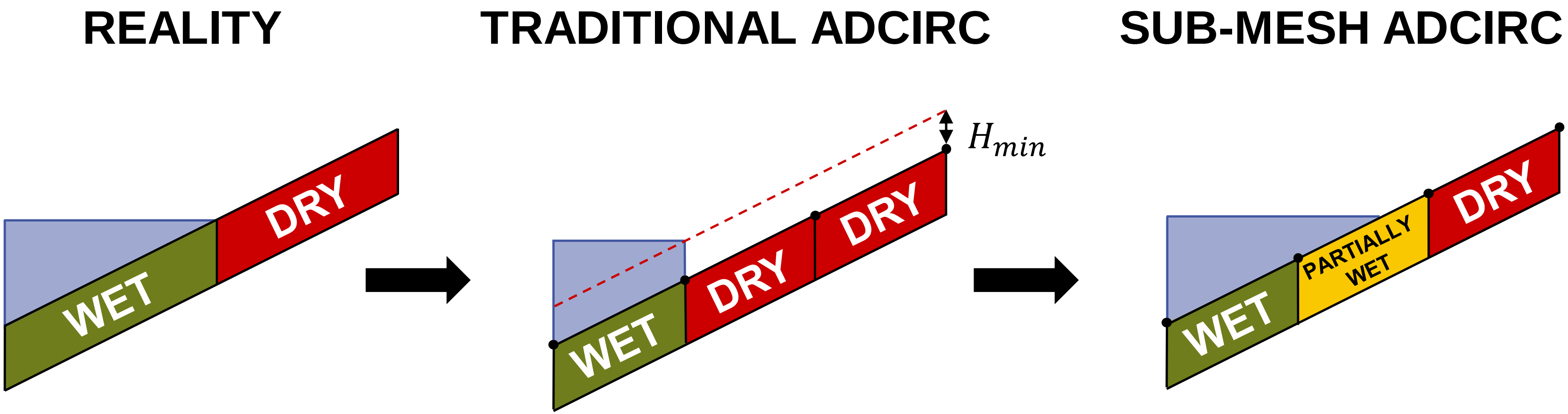
- Divide each sub-area into 2 triangles





- Refine each triangle into many smaller triangles that are roughly the same resolution as the sub-mesh
- Apply DEM data to the smaller triangles
- Average sub-mesh variables over each sub-area

Wetting and Drying



Wetting and Drying Algorithm

- A totally new wetting and drying algorithm was used to replace the existing.
- The wet/dry state of each vertex is determined based on the wet area surrounding the vertex.
- Elemental wet/dry state is determined by the average wet area within each element.

Wetting and Drying Algorithm

BEGIN
Wetting and Drying

→

ADJUST
ζ of **DRY** vertices to the $\bar{\zeta}$ of the neighboring **WET** vertices

→

COMPUTE
averaged variables ϕ, $\langle H \rangle_G$, $\langle H \rangle_W$

→

EVALUATE
wet/dry state
$\phi \geq \phi_{dry}$

→

END
Wetting and Drying

Discretization

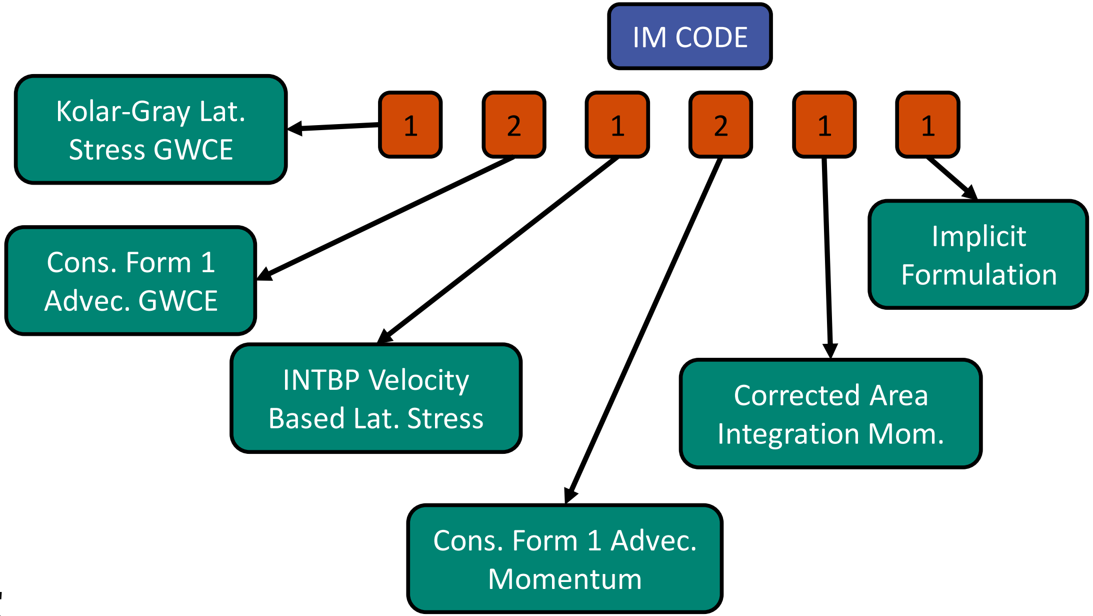
- The discretization of the governing equations mimics the Conservative Version 1 discretization.
- Conservative Version 1 assumes the un-differentiated terms are elementally averaged and uses a linear expansion in space for the conservative flux terms.

Results

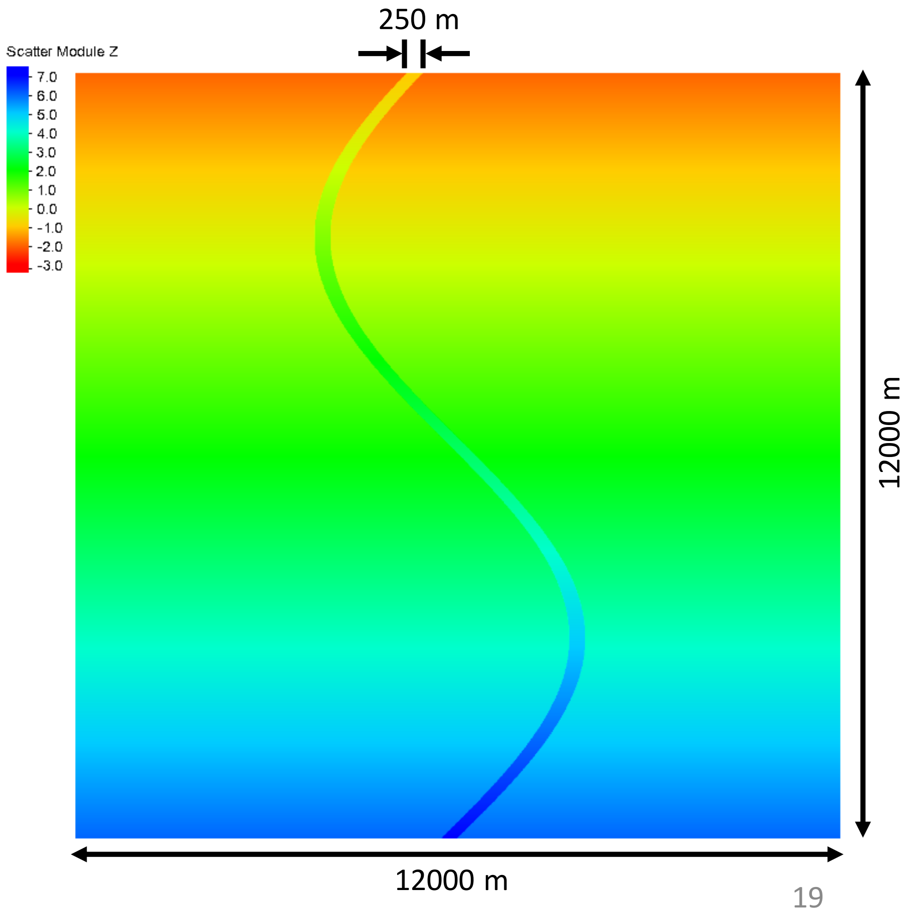
- Sub-mesh ADCIRC was run on a coarsened mesh to demonstrate its ability to accurately predict water surface elevations.
- Traditional ADCIRC was run on both a coarsened and high resolution mesh as a baseline to compare sub-mesh results.

Model Setup

- DT = 1 second
- τ_0= 0.005
- FFACTOR = 0.0025
- ESLM = 2.0
- No ramp
- 1 day runtime
- Sub-mesh $\phi_{dry} = 0.25$
- 2 m diurnal tidal signal was used to force the model

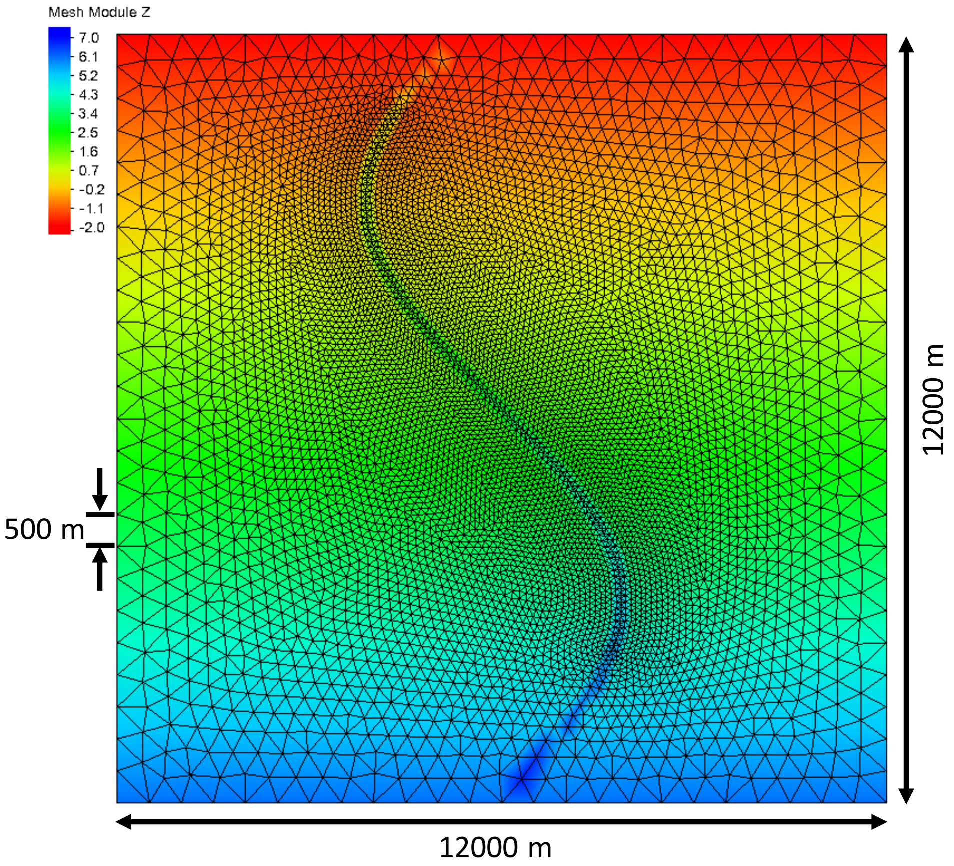


- High resolution (10 m) DEM featuring a 250 m wide winding channel was used to calculate the sub-mesh variables.



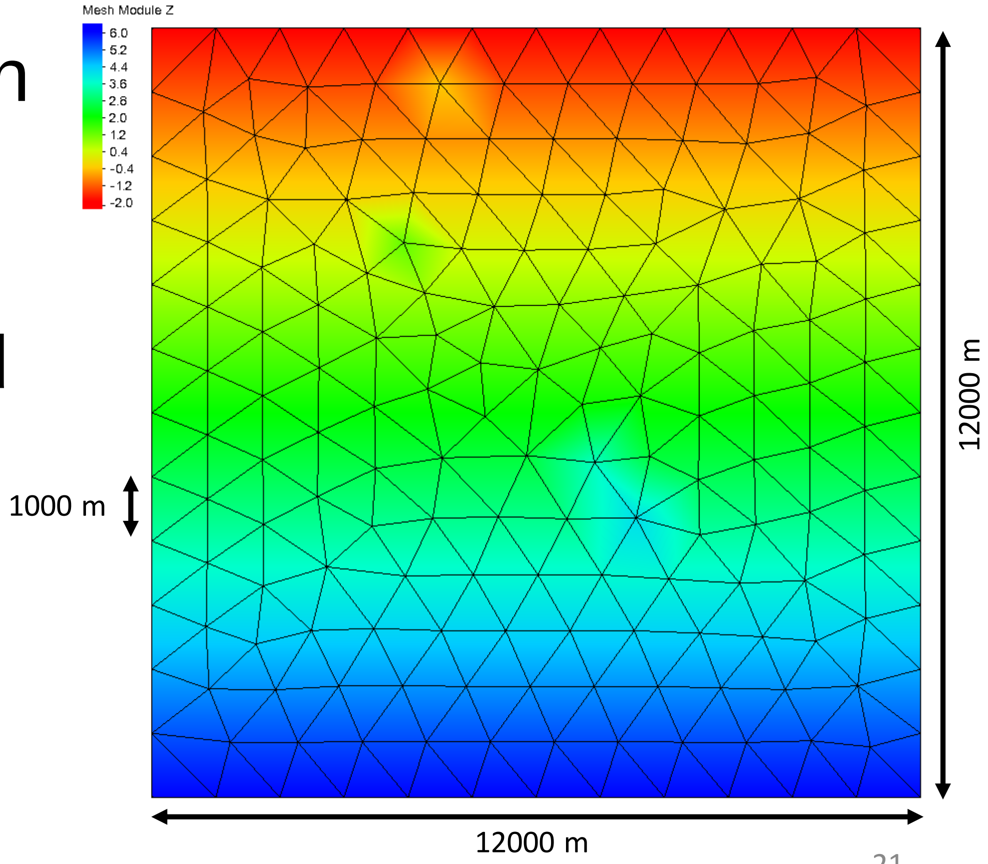
- High resolution mesh was created using this DEM.
- This mesh resolves the small channel.

Vertices: 6420 Elements: 12742

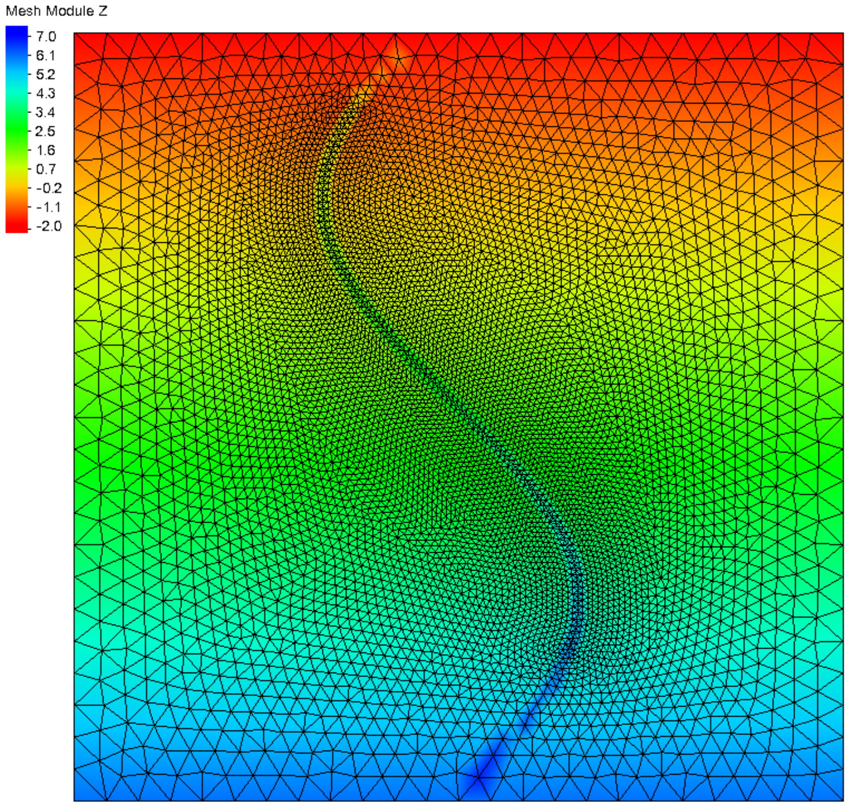


- Coarse resolution mesh was created using the DEM.
- This mesh was created by simply paving elements over the domain.

Vertices: 192 Elements: 334

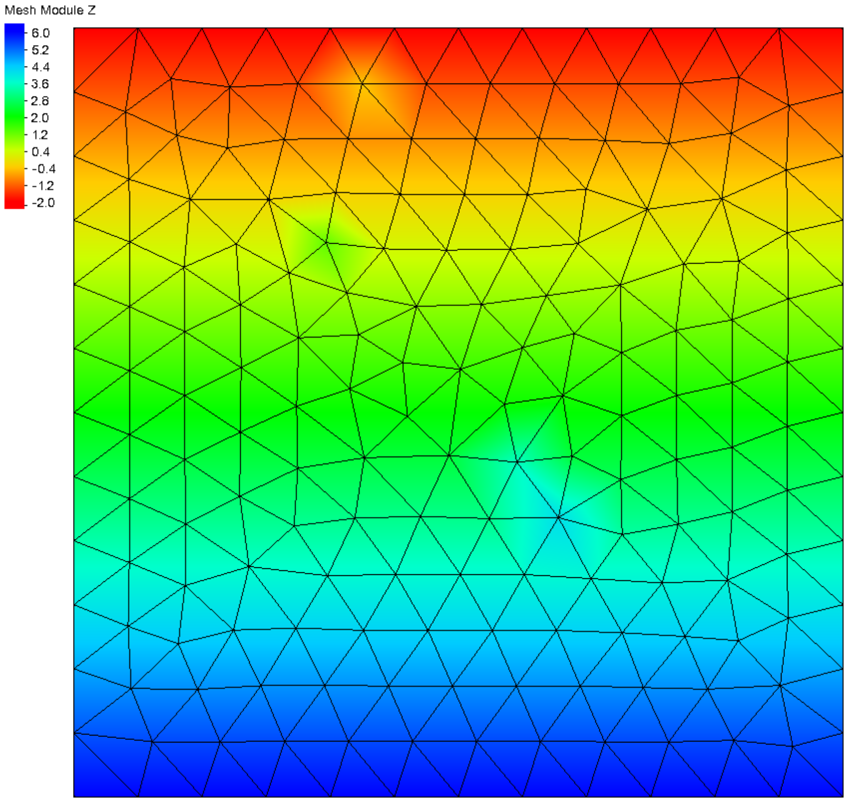


High Resolution

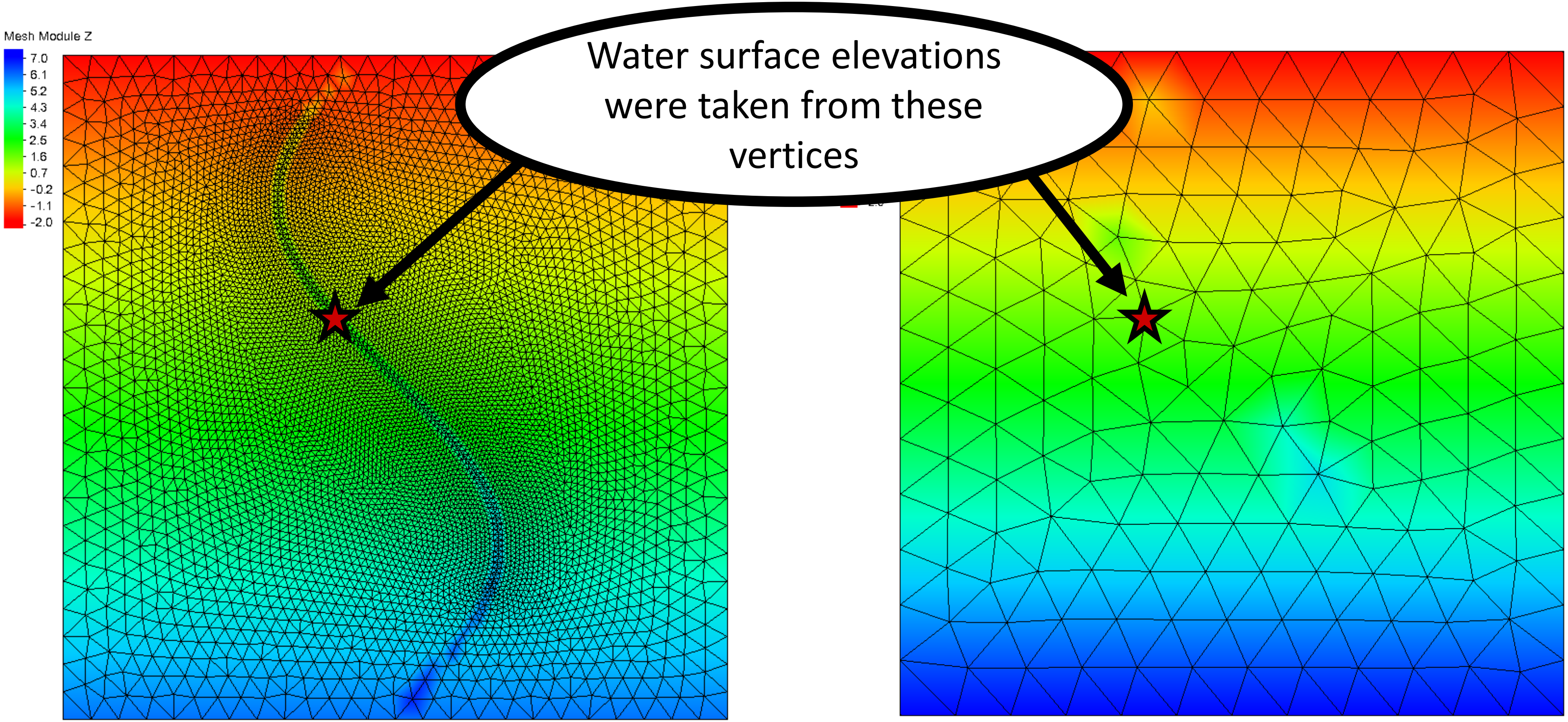


Vertices: 6420 Elements: 12742

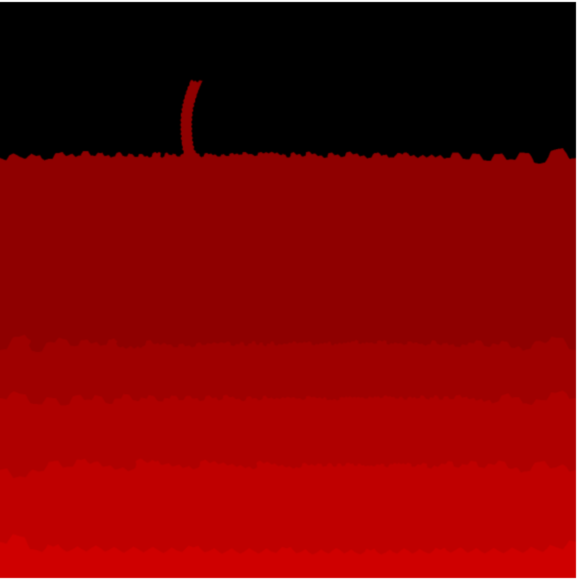
Coarse Resolution



Vertices: 192 Elements: 334

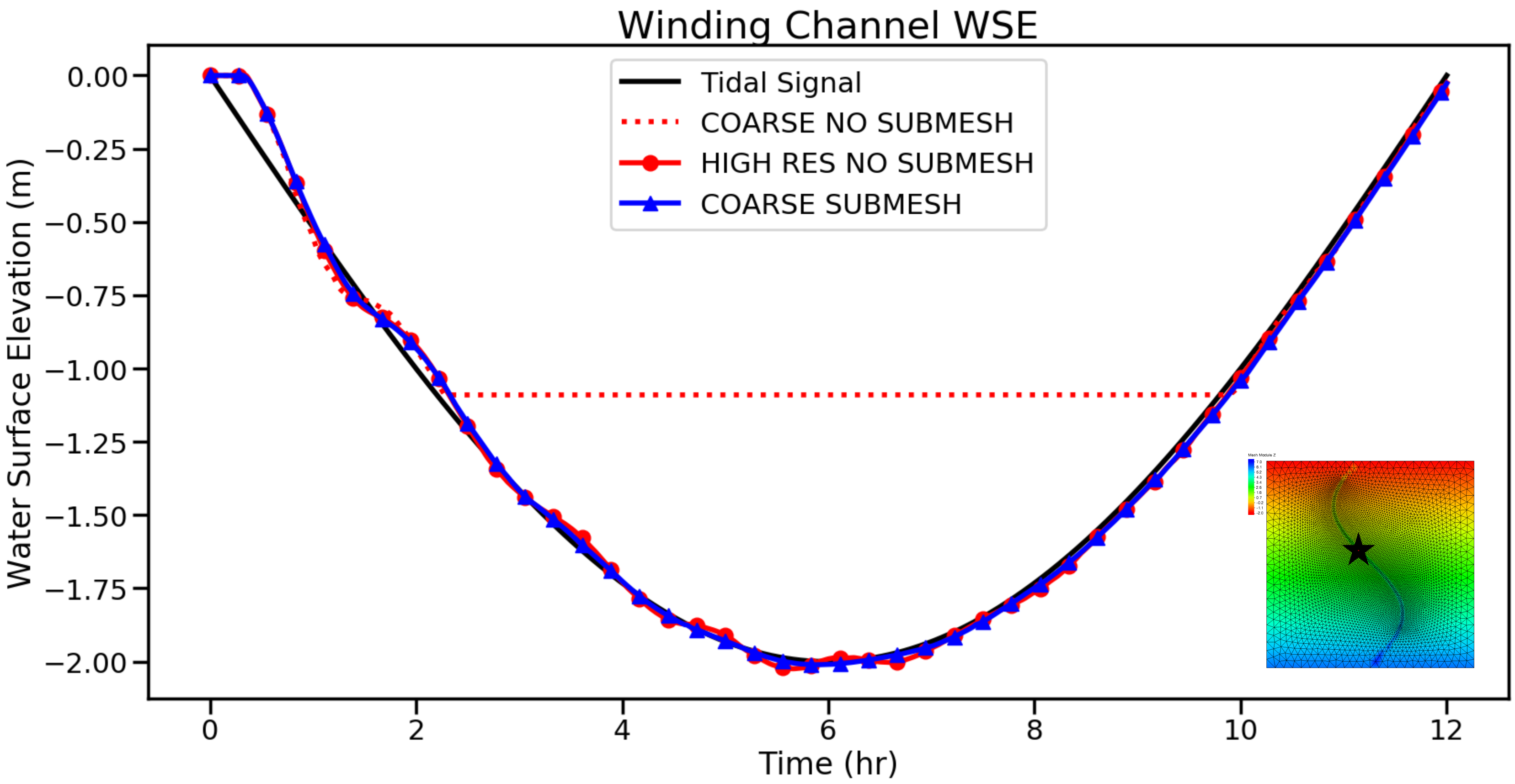

Mesh Module Z
7.0
6.1
5.2
4.3
3.4
2.5
1.6
0.7
-0.2
-1.1
-2.0
Water surface elevations were taken from these vertices

High Resolution



Coarse No SUB-MESH

Coarse SUB-MESH


Winding Channel WSE
Tidal Signal
COARSE NO SUBMESH
HIGH RES NO SUBMESH
COARSE SUBMESH
Water Surface Elevation (m)
Time (hr)
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−2.00
0
2
4
6
8
10
12

RUN TIMES	
High Resolution (no sub-mesh)	591 seconds
Coarse Resolution (no sub-mesh)	14.5 seconds
Coarse Resolution (sub-mesh)	20.75 seconds

- The sub-mesh adds roughly 40% to computation time.
  - This is likely to improve as the code is clean up for use on more realistic test cases.

Conclusion

- The sub-mesh code was able to accurately predict water surface elevation on a coarsened mesh.
- The computational cost of the sub-mesh model was close to 30 times less than the high resolution traditional run.
- Coarse mesh creation was extremely fast and easy compared to creating the high resolution mesh.

Future Work

- The wetting and drying routine is still being adjusted to allow for smaller ϕ_{dry}.
- A threshold of around 5% would be ideal in the future.
- The model is also experiencing stability problems at the top of the domain when running multi-day simulations.
- Testing on a realistic domain such as Caernarvon Marsh, LA is soon to come.

Thank you!

Questions?

Thank you to the National Science Foundation for sponsoring this research